This paper was presented as part of the main technical program at IEEE INFOCOM 2011

Multi-hop Access Pricing in Public Area WLANs

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***Abstract*—Public area WLANs stand for WLANs deployed in public areas such as classroom and office buildings to provide Internet connections. Nevertheless, such a service may not be always available because of limited AP coverage, poor signal strength, or password authentication. Multi-hop access is a feasible approach to facilitate users without direct AP accesses to resort to other online users as relays for data forwarding. This paper employs credit-exchange for multi-hop access in public area WLANs to encourage users to cooperate, and proposes a complete pricing framework. We first investigate a revenue model to define the profit of a relay. Next we point out that cutoff bandwidth allocation is a crucial issue in pricing strategy. Optimal bandwidth allocation schemes are then proposed for two bandwidth demand models. Following that we consider a more practical scenario where the relay's bandwidth capacity and the client's bandwidth demand are bounded, and propose two heuristic algorithms SRMC and MRMC to compute bandwidth allocation and/or relay-client association. Extensive simulation study has been performed to validate our design.**

I. INTRODUCTION

Public area WLANs stand for WLANs deployed in public areas such as classroom and office buildings. They are common wireless access networks that can provide Internet connections to people entering their coverage areas. However, such a service may not be always available. For example, when a person enters a conference room with his laptop, he may find that either the room is not covered by any AP directly, or all the APs are of poor signal strength or protected by password authentication. Multi-hop access is a feasible approach for such a problem as a client may first connect to another online client, and then use that client as a relay to forward its data.

A big challenge of multi-hop access is how to encourage users to cooperate with each other. Felegyhazi et al. [1] point out that forwarding cooperation can exist without incentive mechanisms under some strict conditions. But generally speaking, incentive mechanisms are necessary to encourage users to forward for others. Ad Hoc-VCG [2] pays to the intermediate nodes a premium over their costs for forwarding data packets. Incentive mechanisms can be either *credit-exchange based* such as [3], [4], in which realistic or virtual money is employed, or *reputation based* such as [5], [6], in which each node builds a positive reputation for itself by cooperating with others and is tagged as misbehaving otherwise.

A public area WLAN often has a relatively fixed user group such as students of a classroom building. A user has direct network access sometimes while has to rely on others' relay service at other times. Therefore, if users cooperate with each other, everyone in the group can benefit from a long-term point of view. In this paper a concept of "relay-union" is introduced, which refers to a union of clients who often come to the same public area and are willing to do forwarding for each other. At any moment, a fraction of union members who have direct AP connections play the role of relays, offering forwarding services to others. Credit-exchange is setup within the union, which regulates how clients should pay credits to their relays for the forwarding services. The relay node can use its earned credit to purchase forwarding services when needed. This mechanism takes the advantages of both credit-exchange based and reputation based incentive mechanisms.

Pricing is a key issue in a relay-union. Falkner et al. [7] gives an overview of various network pricing models. Pricing schemes for access networks proposed in related literatures can be divided into *non-competitive pricing* and *competitive pricing* [8]. The former often employs an optimization based approach to maximize the revenue of service providers [9]–[13]. The later usually adopts game theory to evaluate the optimal pricing strategy for competing users or service providers [14]–[19]. We take a non-competitive pricing approach, in which the relays cooperate to maximize their revenues. As all the members in our relay-union take the role of relays from time to time, all users mutually benefit each other from providing relay services in long-term. To our best knowledge, this study is the sole work targeting a relay-union in public area WLANs.

This paper proposes a complete pricing framework that consists of revenue and bandwidth models, and the corresponding optimal pricing and bandwidth allocation strategies for multi-hop access in public area WLANs. The objective of the framework is to maximize the profit of relays. Members of a relay-union are equal as a relay of today is probably a client tomorrow. Therefore the more credits a relay obtains from its service, the more bandwidth it could purchase when it becomes a client in the future. Thus profit maximization is a reasonable target for all members of the relay-union.

We first investigate a revenue model that includes a client price function and a relay cost function, base on which the profit of a relay can be defined. Next we point out that cutoff bandwidth allocation is a crucial issue in pricing strategy by analyzing the relationship between the bandwidth and the profit. Optimal bandwidth allocation schemes are then pro-

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posed for two bandwidth demand models. Following that we consider a more practical scenario where the relay's bandwidth capacity and the client's bandwidth demand are upper and lower bounded, respectively. We then propose two heuristic algorithms, with one targeting single-relay multi-client (SRMC) for bandwidth allocation, and the other targeting multi-relay multi-client (MRMC) for both client-relay association and bandwidth allocation. Extensive simulation study has been performed, and the results indicate that our pricing framework can significantly help to improve the average relay profit and per-user bandwidth.

The rest of the paper is organized as follows. Section II introduces the pricing model and the related definitions. The pricing scheme for the single-relay single-client, single-relay multi-client, and multi-relay multi-client are investigated in Sections III, IV, V, respectively. Simulation study is reported in Section VI. We conclude our paper with Section VII.

II. Models and Definitions

A client should pay credits to the relay who offers forwarding service to it. The pricing mechanism is a key issue in this paper. The service that the client obtains is the most important factor related with pricing. We employ access bandwidth of a client as a metric to measure the service it receives. Note that in our analysis, we focus on "one unit time" whenever an "amount" is involved for simplicity. In other words, we intend to investigate a "pricing model" for a unit time. For example, the price function of a client defined in Definition 2 refers to the amount of charge "per unit time".

Definition 1 (Access bandwidth of a client): Define the access bandwidth of a client to be the amount of data transmitted by the client per unit time.

It is reasonable for a client to pay the forwarding service according to the amount of access bandwidth it receives. However, the relationship between access bandwidth and credit payment should not be a simple linear function. Here we introduce a price function to describe such relationship.

Definition 2 (Price function of a client): Denote by $f_i(B)$ the price function of a client c_i, where B is the client's access bandwidth obtained from its relay and $f_i(B)$ is the upper bound of credits that the client is willing to pay for such a bandwidth.

An example of a client's pricing function $f(B)$ is illustrated in Fig. 1.

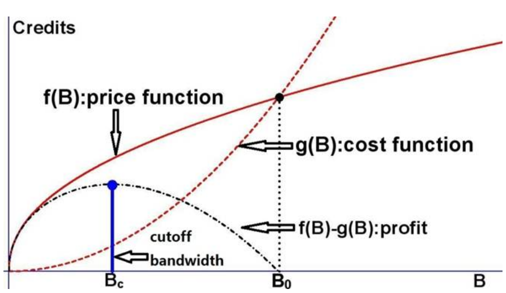


Fig. 1. Price and cost functions

Assumption 1: The price function of a client is concave.

Assumption 1 can be justified as follows. The derivation function $f_i{}'(B)$ of $f_i(B)$ is the marginal utility (MU) of the access bandwidth, which refers to the increase rate of credits that the client is willing to pay relative to the access bandwidth. According to the Law of Diminishing Marginal Utility in economics theory [20], MU is decreasing with the increase of access bandwidth because a client's desire of getting more access bandwidth does not increase with each additional unit of bandwidth acquired. So $f_i{}'(B)$ is a decreasing function. Thus $f_i(B)$ should be concave.

Definition 3 (Serving bandwidth of a relay): Define the serving bandwidth of a relay to be the total amount of bandwidth in bytes that the relay employs for its forwarding service.

A relay's serving bandwidth is the summation of access bandwidths of all its clients. A relay can earn more credits by increasing its serving bandwidth. However, the forwarding service may have negative impact on its own data transmission. Meanwhile, the energy consumption and CPU utilization become heavier with the increase of serving bandwidth. Therefore, a relay not only receives credits but also pays a cost when providing a forwarding service.

Definition 4 (Cost function of a relay): Denote the cost function of a relay r_j by $g_j(B)$, where B is the relay's serving bandwidth and $g_j(B)$ is its cost for offering such a bandwidth.

An example cost function $g(B)$ of a relay is illustrated in Fig. 1.

Assumption 2: The cost function of a relay is convex.

Assumption 2 can be justified as follows. The derivative function $g_j{}'(B)$ of $g_j(B)$ is the marginal cost (MC) of the serving bandwidth, which refers to the increase rate of the cost over the serving bandwidth. MC increases with the increase of the relay's serving bandwidth. The reason is that when the occupancy rate of the CPU or the bandwidth is higher, forwarding for others can cause a greater harm to the performance of the relay. Thus the more the relay's serving bandwidth, the more severe the negative impact it has on the performance of the relay. Therefore $g_j{}'(B)$ is an increasing function. Thus $g_j(B)$ is assumed to be convex.

The price function of a client can be evaluated by analyzing and curve-fitting history data. The same goes for the cost function of a relay.

Definition 5 (Charge function): For a relay r_j and a client c_i, let $h_{ij}(B)$ denote the charge function between them, where B is the access bandwidth of c_i and $h_{ij}(B)$ is r_j's charge to c_i, which is the amount of credits that c_i should pay to r_j for the access bandwidth B.

Having the charge function, a relay's total charge can be evaluated by accumulating the charges of all clients. Relays want to maximize their profit, which is defined as follows.

Definition 6 (Profit of a relay): The profit P_j of a relay r_j is defined to be the difference between its total charge and its cost for its serving bandwidth.

III. Pricing for Single-Relay Single-Client

This section addresses the pricing strategy when there is a single relay r_j serving a single client c_i.

The relay r_j wants to charge a $h_{ij}(B)$ that is as high as possible. The $h_{ij}(B)$ should be at least higher than $g_j(B)$ to ensure that r_j can make benefit rather than suffer loss. Nevertheless, the charge should not exceed $f_i(B)$ since otherwise, the client would reject the service. Fig. 1 illustrated a relay's profit $f_i(B) - g_j(B)$. The x axis indicates both the client's access bandwidth and the relay's serving bandwidth because there is only one client. When $B < B_0$, $f_i(B) > g_j(B)$. Thus the relay can employ $f_i(B)$ as its charge function to maximize its profit. Therefore we have $h_{ij}(B) = f_i(B)$ for $B < B_0$.

It can be observed from Fig. 1 that the profit of a relay begins to decline if the serving bandwidth exceeds a threshold. Therefore, the relay should limit the client's maximum access bandwidth by a cutoff bandwidth.

Definition 7 (Client cutoff bandwidth): Define the client cutoff bandwidth of a client c_i, denoted by B_c^i [1], to be the maximum access bandwidth it is allowed to use, which is limited by its relay.

Definition 8 (Relay cutoff bandwidth B_{cr}^j): Define the relay cutoff bandwidth of a relay r_j, denoted by B_{cr}^j, to be the summation of its client cutoff bandwidths.

A relay should set the client cutoff bandwidth to an optimal value in order to maximize its profit. In the case of one client c_i and one relay r_j, such an optimal B_c^i should satisfy $f_i(B_c^i) - g_j(B_c^i) \geq f_i(B) - g_j(B)$ for $\forall B$. For example, the B_c in Fig. 1 is the optimal client cutoff bandwidth that maximizes the relay's profit. In the following we discuss how to evaluate the cutoff bandwidth.

Definition 9 (Critical MU): Define the critical MU of a client c_i to be ${f_i}'(B_c^i)$, which is the marginal utility of its price function at its client cutoff bandwidth.

Definition 10 (Critical MC): Define the critical MC of a relay r_j to be ${g_j}'(B_{cr}^j)$, which is the marginal cost of its cost function at its relay cutoff bandwidth.

Theorem 1: Under single-relay single-client scenario, the profit of relay r_j is maximized when the critical MU of its client c_i equals the critical MC of itself, which means that ${f_i}'(B_c^i) = {g_j}'(B_{cr}^j)$ holds.

Proof 1: The relay's profit $P_j = f_i(B) - g_j(B)$ should be maximized when $B = B_c^i$. Therefore B_c^i is the zero-point of the derivative function of P_j, which means ${f_i}'(B_c^i) = {g_j}'(B_c^i)$. Since there is only one client, we have $B_c^i = B_{cr}^j$. Thus ${f_i}'(B_c^i) = {g_j}'(B_{cr}^j)$. ■

The intuitive meaning of this theorem is obvious. As shown in Fig. 1, when a client's access bandwidth is lower than its cutoff bandwidth, its MU ${f_i}'(B)$ is larger than the relay's MC ${g_j}'(B)$. At this time the relay's charge grows more quickly than its cost, which means that the relay's profit grows with the increase of the serving bandwidth. However, if the access bandwidth exceeds the cutoff bandwidth, at which ${f_i}'(B) < {g_j}'(B)$, the relay's charge grows more slowly than its cost. Thus the relay's profit goes down with the increase of the serving bandwidth. In other words, ${f_i}'(B)$ is a decreasing function while ${g_j}'(B)$ is increasing, and their intersection point is just the critical MU/critical MC.

[1]For bandwidth notations such as B_c, B_{cr}, B_{min} and B_{max}, index is the superscript, i.e, B_c^i. But for function notations such as f, g, h, q, index is the subscript, i.e., f_i.

IV. Pricing for Single-Relay Multi-Client

This section addresses the pricing strategy of relay r_j when it is connected to N clients, denoted by $C = \{c_1, c_2, ..., c_N\}$. To maximize its profit, r_j should still bid a charge function that is equal to the client's pricing function. But how to determine the set of client cutoff bandwidths, denoted by $\{B_c^i\} = \{B_c^1, B_c^2, ...B_c^N\}$, becomes a complex issue.

Similar to the case of single client, the relay's profit first increases then decreases with the increase of its serving bandwidth. However, the profit cannot be determined solely by serving bandwidth. It is also related to the access bandwidth of each client that is determined by its bandwidth demand.

Definition 11 (Bandwidth demand of a client): Define the bandwidth demand of a client to be the bandwidth required for its current data transmission.

A. Infinite bandwidth demand model

In this subsection, we consider an ideal bandwidth demand model, in which the bandwidth demand of every client is infinite. This means that a client always has infinite amount of data for transmission. Thus a client's access bandwidth is equal to its cutoff bandwidth offered by its relay. Therefore, the relay can evaluate an optimal client cutoff bandwidth allocation to maximize its profit without considering the bandwidth demands of its clients. The profit P of the relay can be calculated by the difference between its total charge $\sum_{i=1}^{N} f_i(B_c^i)$ and its total cost $g(B_{cr})$:

$$P = \sum_{i=1}^{N} f_i(B_c^i) - g(B_{cr}), \tag{1}$$

where $B_{cr} = \sum_{i=1}^{N} B_c^i$, which is the relay cutoff bandwidth. Then the optimal client cutoff bandwidth allocation $\{B_c^i\} = \{B_c^0, B_c^1, ..., B_c^N\}$ to maximize the relay's profit should be the solution to the following problem.

$$\begin{aligned} &\max_{\{B_c^i\}} P \\ &s.t. \quad B_c^i \geq 0 \end{aligned} \tag{2}$$

Theorem 2: Under infinite bandwidth demand model and optimal client cutoff bandwidth allocation, the critical MU of every client equals the critical MC of the relay.

Proof 2: To solve Eq. (2), we need to evaluate the maximum value of the multivariate function $P(\{B_c^i\}) = \sum_{i=1}^{N} f_i(B_c^i) - g(B_{cr})$. Thus we calculate the partial derivative function of P for each B_c^i and set it to 0. From $\frac{\partial \sum_{i=1}^{N} f_i(B_c^i)}{\partial B_c^i} = \frac{df_i(B_c^i)}{dB_c^i}, 1 \leq i \leq N$ and $\frac{\partial g(B_{cr})}{\partial B_c^i} = \frac{dg(B_{cr})}{dB_{cr}} \times \frac{dB_{cr}}{dB_c^i} =$

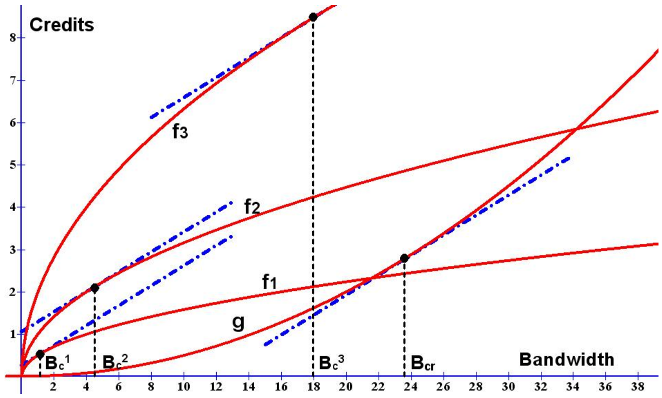


Fig. 2. An illustration of Theorem 2

$\frac{dg(B_{cr})}{dB_{cr}}, 1 \leq i \leq N$, we get the following set of equations.

$$\begin{aligned} \frac{df_i(B_c^i)}{dB_c^i} &= \frac{dg(B_{cr})}{dB_{cr}}, 1 \leq i \leq N \\ s.t. \quad B_c^i &\geq 0 \end{aligned} \tag{3}$$

Since $\frac{df_i(B_c^i)}{dB_c^i}$ is the critical MU of the client c_i and $\frac{dg(B_{cr})}{dB_{cr}}$ is the critical MC of the relay, the theorem holds. ■

Let's take a look at a simple example of three clients with price functions $f_i(B) = a_i B^{\frac{1}{2}}, 1 \leq i \leq 3$ and a relay cost function $g(B) = bB^2$, where a_i and b are parameters. Then Eq. (3) can be written as

$$\begin{cases} \frac{1}{2}a_1(B_c^1)^{-\frac{1}{2}} = 2b(B_c^1 + B_c^2 + B_c^3) \\ \frac{1}{2}a_2(B_c^2)^{-\frac{1}{2}} = 2b(B_c^1 + B_c^2 + B_c^3) \\ \frac{1}{2}a_3(B_c^3)^{-\frac{1}{2}} = 2b(B_c^1 + B_c^2 + B_c^3) \end{cases} \tag{4}$$

By solving Eq. (4) we obtain the expressions shown in the middle column of Table I. The numerical results are reported in the third column when $a_1 = 0.5$, $a_2 = 1$, $a_3 = 2$, and $b = 0.005$. Fig. 2 is the corresponding image of this example.

Note that Fig. 2 illustrates the intuitive meaning of Theorem 2. We obtain four tangent lines with the tangency points located at $(B_c^1, f_1(B_c^1))$, $(B_c^2, f_2(B_c^2))$, $(B_c^3, f_3(B_c^3))$ and $(B_{cr}, g(B_{cr}))$. These four tangent lines are parallel to each other, which means that $f_1{}'(B_c^1) = f_2{}'(B_c^2) = f_3{}'(B_c^3) = g'(B_{cr})$, demonstrating the equivalence of critical MUs and the critical MC. Meanwhile, $B_c^1 + B_c^2 + B_c^3 = B_{cr}$ because B_{cr} is the relay cutoff bandwidth. Note that B_c^1, B_c^2, and B_c^3 are the optimal client cutoff bandwidths that can maximize the profit of the relay.

TABLE I
EVALUATION RESULT OF THE EXAMPLE IN SUBSECTION A

Parameters	Expressions	Values
B_c^1	$\frac{a_1^2}{\sqrt[3]{16b^2(a_1^2+a_2^2+a_3^2)^2}}$	1.1233
B_c^2	$\frac{a_2^2}{\sqrt[3]{16b^2(a_1^2+a_2^2+a_3^2)^2}}$	4.4930
B_c^3	$\frac{a_3^2}{\sqrt[3]{16b^2(a_1^2+a_2^2+a_3^2)^2}}$	17.972
B_{cr}	$\sqrt[3]{\frac{a_1^2+a_2^2+a_3^2}{16b^2}}$	23.588
Critical MU	$\frac{\sqrt[3]{4b(a_1^2+a_2^2+a_3^2)}}{2}$	0.2359
Critical MC	$\frac{\sqrt[3]{4b(a_1^2+a_2^2+a_3^2)}}{2}$	0.2359
P	$\frac{(a_1^2+a_2^2+a_3^2)}{\sqrt[3]{4b(a_1^2+a_2^2+a_3^2)}} - b(a_1+a_2+a_3)^2$	10.991

B. Dynamic bandwidth demand model

We want to consider a more practical bandwidth demand model. In realistic networks, the bandwidth demand of a client should variate in time because the client does not always have infinite amount of data to transmit at any instant of time. Different clients may have different bandwidth demands because of different application types running on them.

For a client c_i, assume that its dynamic bandwidth demand is a continuous random variable which follows a certain distribution with a probability density function $q_i(B)$. The client tells the relay about its $q_i(B)$, and the relay replies with the client cutoff bandwidth B_c^i. If the client's bandwidth demand exceeds B_c^i during its data transmission, its access bandwidth is restricted to the client cutoff bandwidth.

Under this bandwidth demand model, the charge, cost, and profit of the relay should be evaluated from the perspective of mathematical expectations. The expectation of the relay's charge from a client c_i is $\int_0^{B_c^i} f_i(B)q_i(B)dB + \int_{B_c^i}^{+\infty} f_i(B_c^i)q_i(B)dB$.Thus the expectation of the relay's total charge can be expressed by Eq. (5).

$$E_{charge} = \sum_{i=1}^{N}(\int_0^{B_c^i} f_i(B)q_i(B)dB + \int_{B_c^i}^{+\infty} f_i(B_c^i)q_i(B)dB) \tag{5}$$

Then Eq. (6) gives the expectation of the relay's serving bandwidth.

$$E_{bs} = \sum_{i=1}^{N}(\int_0^{B_c^i} Bq_i(B)dB + \int_{B_c^i}^{+\infty} B_c^i q_i(B)dB) \tag{6}$$

Thus the expectation of the relay's total cost can be evaluated as Eq. (7).

$$E_{cost} = g(E_{bs}) \tag{7}$$

That is to say, the expectation of the relay's profit is

$$E_p = E_{charge} - E_{cost}. \tag{8}$$

Then an optimal client cutoff bandwidth allocation that can maximize the expectation of the relay under the dynamic bandwidth demand model should be the solution of the following problem.

$$\begin{aligned} &\max_{\{B_c^i\}} E_p \\ &s.t. \quad B_c^i \geq 0 \end{aligned} \tag{9}$$

The definition of the relay's critical MC should also be extended from the perspective of mathematical expectations, i.e., $g'(E_{bs})$ instead of $g'(B_{cr})$. We have

Theorem 3: Under the dynamic bandwidth demand model and the optimal client cutoff bandwidth allocation, the critical MU of every client equals the critical MC of the relay.

TABLE II
NUMERICAL RESULT OF THE EXAMPLE IN SUBSECTION B

B_c^1	0.1826	B_c^2	0.7306
B_c^3	2.9223	E_{bs}	2.9249
$CriticalMU$	0.5850	$CriticalMC$	0.5850
P	3.5107		

Proof 3: The solution approach to Eq. (9) is similar to that of Eq. (2). We set the partial derivative function of E_p for each B_c^i to be zero, i.e. $\frac{\partial E_p}{\partial B_c^i} = \frac{\partial E_{charge}}{\partial B_c^i} - \frac{\partial E_{cost}}{\partial B_c^i} = 0$, $1 \le i \le N$. From $\frac{\partial E_{charge}}{\partial B_c^i} = \frac{df_i(B_c^i)}{dB_c^i}\int_{B_c^i}^{+\infty} q_i(B)dB$ and $\frac{\partial E_{cost}}{\partial B_c^i} = \frac{dg(E_{bs})}{dE_{bs}}\int_{B_c^i}^{+\infty} q_i(B)dB$, we get Eq. (10).

$$\begin{aligned} \frac{df_i(B_c^i)}{dB_c^i} &= \frac{dg(E_{bs})}{dE_{bs}}, 1 \le i \le N \\ s.t. \quad B_c^i &\ge 0 \end{aligned} \tag{10}$$

The solution $B_c^i = \{B_c^0, B_c^1, ..., B_c^N\}$ of Eq. (10) is just the optimal bandwidth allocation. It is clear that the critical MU $\frac{df_i(B_c^i)}{dB_c^i}$ of each client c_i equals the critical MC of the relay $\frac{dg(E_{bs})}{dE_{bs}}$. ■

The simple example in subsection A is employed again to illustrate the cutoff bandwidth allocation under the dynamic bandwidth demand model. Suppose that bandwidth demand of each client follows a uniform distribution in $[c, d]$. Then the probability density function $q_i(B) = \frac{1}{d-c}, c \le B \le d, i = 1, 2, 3$. We obtain Eq. (11) from Eq. (10):

$$\begin{cases} \frac{a_1}{2(B_c^1)^{\frac{1}{2}}} = \frac{b}{c-d}\sum_{i=1}^{3}((B_c^i)^2 - 2dB_c^i + c^2) \\ \frac{a_2}{2(B_c^2)^{\frac{1}{2}}} = \frac{b}{c-d}\sum_{i=1}^{3}((B_c^i)^2 - 2dB_c^i + c^2) \\ \frac{a_3}{2(B_c^3)^{\frac{1}{2}}} = \frac{b}{c-d}\sum_{i=1}^{3}((B_c^i)^2 - 2dB_c^i + c^2) \end{cases} \tag{11}$$

The analytic expressions of the client cutoff bandwidths are quite complex. Therefore we only give the numerical results in Table II when $a_1 = 0.5, a_2 = 1, a_3 = 2, b = 0.1, c = 0$, and $d = 5$. In fact, thanks to various numerical algorithms and mathematical softwares, it is not difficult to solve Eq. (3) and (10) even when f and g are very complicated.

C. Capacity limitation and minimum bandwidth demand

The above investigation of the relay's pricing strategy implies an assumption that the capacity of a relay is unlimited and a client can accept an infinitesimally small bandwidth. In reality, a relay's capacity is always upper-bounded and a client's bandwidth demand is always lower-bounded. Therefore a relay can not afford to serve an excessive number of clients and a client can not accept a relay that could not supply a minimum bandwidth. Denote by B_{max} the maximum serving bandwidth of the relay and by B_{min}^i the minimum bandwidth demand of client c_i. These two new restrictions make our model more practical. The problem of optimal client cutoff bandwidth allocation can be formulated by:

$$\begin{aligned} &\max_{\{B_c^i\}} P \\ s.t. \quad &B_c^i \ge B_{min}^i \\ &B_{cr} \le B_{max}. \end{aligned} \tag{12}$$

Eq. (12) is used for infinite bandwidth demand model. If we change P to E_p, it is also fit for dynamic bandwidth demand model. We conjecture that Eq. (12) implies a NP-hard problem, which will be investigated in our future research. Thus we design an approximate algorithm named SRMC (Single-Relay Multi-Client) to compute a feasible cutoff bandwidth allocation.

Algorithm 1: SRMC

Input: B_{max}, $C = \{c_1, c_2, ..., c_N\}$, $\{B_{min}^1, B_{min}^2, ..., B_{min}^N\}$, r, $\{f_1, f_2, ..., f_N\}$, g
Output: $\{B_c^i\}$
$B_{cr} \leftarrow Infinity$
while $B_{cr} > B_{max}$ **do**
  // Optimal allocation
  $\{B_c^i\} \leftarrow BandAlloc(r, C)$
  // Ensure minimum bandwidth demand of every client
  for $i = 1; i \le N; i++$ **do**
    if $B_c^i < B_{min}^i$ **then**
      $B_c^i \leftarrow B_{min}^i$
    end
  end
  $B_{cr} \leftarrow \sum_{i=1}^{N} B_c^i$
  if $B_{cr} \le B_{max}$ **then**
    break
  end
  // Select the client c_{index} with the lowest contribution
  for *each* c_i *in* C **do**
    $\delta_i \leftarrow ComputeDelta(c_i)$
  end
  $index \leftarrow \mathbf{arg\,min}_i\, \delta_i, \forall c_i \in C$
  // Stop serving c_{index}
  $B_{cr} \leftarrow B_{cr} - B_c^{index}$
  $B_c^{index} \leftarrow 0$
  $C \leftarrow C - \{c_{index}\}$
end
return$\{B_c^i\}$

SRMC is a greedy algorithm with B_{max}, the set of clients C, the minimum bandwidth demand of each client, the relay r, the price functions of clients, and the cost function g of the relay as its inputs. Its output is the client cutoff bandwidth allocation $\{B_c^i\}$. The design motivation is stated as follows. If the relay can not afford to serve all the clients while keeping its profit as high as possible, the clients with low contributions to the relay's profit are not served. Here we introduce a metric δ_i to quantify c_i's contribution to the relay's profit.

$$\delta_i = f_i(B_c^i) - (g(B_{cr}) - g(B_{cr} - B_c^i)), \tag{13}$$

where $g(B_{cr} - B_c^i)$ is the part of the relay's cost caused by the clients other than c_i. Thus δ_i stands for c_i's portion in the relay's profit. Eq. (13) is adopted for infinite bandwidth demand model. Under the dynamic bandwidth demand model, a similar expression of δ_i is derived as shown in Eq. (14):

$$\delta_i = E_{charge}^i - (g(E_{bs}) - g(E_{bs} - E_b^i)) \tag{14}$$

where $E^i_{charge} = \int_0^{B^i_c} f_i(B)q_i(B)dB + \int_{B^i_c}^{+\infty} f_i(B^i_c)q_i(B)dB$, $E_{bs} = \sum_{j=1}^{N}(\int_0^{B^j_c} Bq_j(B)dB + \int_{B^j_c}^{+\infty} B^j_c q_j(B)dB)$, and $E^i_b = \int_0^{B^i_c} Bq_i(B)dB + \int_{B^i_c}^{+\infty} B^i_c q_i(B)dB$.

The main body of SRMC consists of a loop to eliminate those clients with low-contribution until $B_{cr} < B_{max}$. At each iteration of the loop, we first call a function $BandAlloc(r, C)$ to compute an optimal bandwidth allocation without considering the constraints of capacity limitation and minimum bandwidth demand. This function takes a relay r and a set of clients C as its input and returns the optimal bandwidth allocation $\{B^i_c\}$ according to Eq. (3) or (10). Next, we check those clients whose cutoff bandwidth is lower than their minimum bandwidth demand, and set their B^i_c to be B^i_{min} to satisfy the minimum bandwidth demand restriction. Then we use a function $ComputeDelta(c_i)$ to calculate δ_i of each client c_i in C according to Eq. (13) or (14). The client with the lowest δ_i is selected and removed from C, and its client cutoff bandwidth is set to be zero. After that, we update B_{cr}. If the capacity limitation is still not satisfied with the new B_{cr}, the elimination process is repeated. Otherwise the algorithm terminates. If there is a bandwidth gap between B_{cr} and B_{max}, the remaining bandwidth can be allocated to the client whose B^i_c is closest to its B^i_{min}.

We denote the time complexity of $BandAlloc()$ by $T_B(N)$, which depends on the method utilized for solving Eq. (3) or (10). $BandAlloc()$ is called at most N times in the main loop of SRMC. Thus the time complexity of SRMC is $T(SRMC) = O(N)T_B(N)$.

V. Pricing for Multi-relay Multi-Client

In this section we study the scenario when there are multiple relays in the network. All relays are members of a relay-union, thus it is reasonable to assume that they are cooperative. Otherwise, relays are likely to cut down their prices to attract more clients, which leads to that profit of relays cannot be maximized and community interest of the relay-union is harmed.

Each relay can still bid a charge function that is equal to the client's pricing function. Then the following two problems need to be investigated: how to associate clients with relays and how to allocate client cutoff bandwidths.

Assume that there are K relays denoted by $R = \{r_1, r_2, ..., r_K\}$. Define by $X = (x_{ij})$ a binary association matrix with $x_{ij} = 1$ if and only r_i serves c_j. Let $B = (b_{ij})$ be a client cutoff bandwidth matrix with b_{ij} being the cutoff bandwidth assigned to c_j by r_i. If infinite bandwidth demand model is utilized, the profit P_i of the relay r_i can be evaluated by

$$P_i = \sum_{j=1}^{N} x_{ij} f_j(b_{ij}) - g_i(\sum_{j=1}^{N} x_{ij} b_{ij}) \qquad (15)$$

The client-relay association and the client cutoff bandwidth allocation problem can be formulated as follows.

$$\begin{aligned}
&\max_{X,B}\{\sum_{i=1}^{K} P_i\} \\
s.t.\quad &\sum_{i=1}^{K} x_{ij} = 1, && 1 \le j \le N \\
&\sum_{j=1}^{N} x_{ij} b_{ij} \le B^i_{max}, && 1 \le i \le K \\
&\sum_{i=1}^{K} x_{ij} b_{ij} \ge B^j_{min}, && 1 \le j \le N \\
&x_{ij} \in \{0, 1\}, \quad 1 \le i \le K \text{ and } 1 \le j \le N
\end{aligned} \qquad (16)$$

In Eq. (16), the association matrix X and the client cutoff bandwidth matrix B are the variables to be computed. The objective function is to maximize the summation of all relays' profits. The first and the last constraints indicate that x_{ij} is a binary variable; the second constraint indicates that the maximum serving bandwidth of the relay r_i can not exceed its capacity limitation B^i_{max}; while the third constraint indicates that the received bandwidth of a client c_j should not be smaller than its minimum bandwidth demand B^j_{min}.

If we consider the dynamic bandwidth demand model, the profit P_i of the relay r_j in Eq. (16) should be replaced by the expected profit E^i_p of r_i.

$$\begin{aligned}
E^i_p &= \textstyle\sum_{j=1}^{N} x_{ij}(\int_0^{B^j_c} f_j(B)q_j(B)dB + \int_{B^j_c}^{+\infty} f_j(B^j_c)q_j(B)dB) \\
&-g_i(\textstyle\sum_{j=1}^{N} x_{ij}(\int_0^{B^j_c} Bq_j(B)dB + \int_{B^j_c}^{+\infty} B^j_c q_j(B)dB))
\end{aligned} \qquad (17)$$

We claim that the problem defined by Eq. (16) is NP-hard. Consider a special case of Eq. (16) in which B is a constant matrix and all price and cost functions are linear. Then Eq. (16) becomes a linear integer programming problem, which is NP complete. Therefore the problem defined by Eq. (16) is at least as hard as integer programming. In the next, we propose an approximate greedy heuristic termed MRMC (Multi-Relay Multi-Client) to compute a feasible solution for Eq. (16).

The inputs of the algorithm include the set of clients C, the set of relays R, the price function of each client, the cost function of each relay, the maximum serving bandwidth of each relay, and the minimum bandwidth demand of each client. The outputs of MRMC are the association matrix X and the client cutoff bandwidth matrix B.

The main idea of MRMC is similar to that of SRMC. We first examine all client-relay pairs to find out the one with the highest contribution to the network profit. Then the client is associated with that relay. Here we use the δ_{ij} defined in Eq. (13) or (14) as the metric to quantify the profit contribution. This process is repeated until all clients are associated with some relays or all relays are at full capacity.

The first step of MRMC is to initialize C_{r_i}, which is the set of clients currently associated with the relay r_i. Then we enter the main while-loop of the algorithm. At each iteration, we select the (c_{index_c},r_{index_r}) pair with the highest $\delta_{index_c,index_r}$

Algorithm 2: MRMC

Input: $C = \{c_1, c_2, ..., c_N\}$, $R = \{r_1, r_2, ..., r_K\}$, $\{f_1, f_2, ..., f_N\}$, $\{g_1, g_2, ..., g_K\}$, $\{B^1_{max}, B^2_{max}, ..., B^K_{max}\}$, $\{B^1_{min}, B^2_{min}, ..., B^N_{min}\}$

Output: $X = (x_{ij})$, $B = (b_{ij})$

// Define C_{r_i} as the set of clients currently served by r_i

$C_{r_i} \leftarrow \Phi, i = 1, 2, ..., K$

$index_r \leftarrow 0, index_c \leftarrow 0$

while $C! = \Phi$ *and* $R! = \Phi$ **do**

  // Update δ and the bandwidth for each (c, r) pair

  for *each* r_i *in* R **do**

    // If $i \neq index_r$, the corresponding δ and the bandwidth allocation does not change compared to the last while-loop

    if $index_r = 0$ *or* $index_r = i$ **then**

      for *each* c_j *in* C **do**

        $\{\delta_{ij}, B_c^{C_{r_i} \bigcup \{c_j\}}\} \leftarrow SRMC(r_i, C_{r_i} \bigcup \{c_j\}, c_j)$

      end

    end

  end

  // Select $(c_{index_c}, r_{index_r})$ with the highest δ

  $\{index_r, index_c\} \leftarrow \mathbf{arg\,max}_{i,j}\, \delta_{ij}, \forall r_i \in R, \forall c_j \in C$

  // Associate c_{index_c} with r_{index_r}

  $x_{index_r, index_c} \leftarrow 1$

  $C \leftarrow C - \{c_{index_c}\}$

  // Update $C_{r_{index_r}}$

  $C_{r_{index_r}} \leftarrow C_{r_{index_r}} \bigcup \{c_{index_c}\}$

  $B_{cr}^{index_r} \leftarrow 0$

  // Update the corresponding bandwidth allocation

  for *each* $c_k \in C_{r_{index_r}}$ **do**

    $b_{index_r, k} \leftarrow B_c^{c_k}$

    $B_{cr}^{index_r} \leftarrow B_{cr}^{index_r} + b_{index_r, k}$

  end

  // A relay at full capacity should not serve any more clients

  if $B_{cr}^{index_r} \geq B_{max}^{index_r}$ **then**

    $R \leftarrow R - \{r_{index_r}\}$

  end

end

return$\{X, B\}$

among all clients in C and all relays in R. Note that it is not necessary to re-compute all the δ_{ij} values except in the first iteration. We only need to update those δ_{ij} values related to the relay r_{index_r} identified in the last iteration. Next we call the algorithm SRMC to evaluate r_i's bandwidth allocation for the clients in $C_{r_i} \bigcup \{c_j\}$ under the constraints of relay capacity limitation and client minimum bandwidth demand. SRMC returns a bandwidth allocation vector $B_c^{C_{r_i} \bigcup \{c_j\}}$, which indicates the cutoff bandwidth of each client in $C_{r_i} \bigcup \{c_j\}$. SRMC also returns the corresponding δ_{ij}. This is achieved by making two modifications to the Algorithm 1: first, c_j should be added to the input of SRMC; second, Line 21 of SRMC should be changed to "return $\{ComputeDelta(c_j), \{B_c^i\}\}$". After that, c_{index_c} is associated with r_{index_r} and removed from C; $C_{r_{index_r}}$ is updated by adding c_{index_c}. The cutoff bandwidth of all clients associated with r_{index_r} is also updated according to the corresponding bandwidth allocation vector previously returned by SRMC. The relay r_{index_r}'s serving bandwidth is also updated. If it is higher than the capacity r_{index_r}, r_{index_r} should be removed from R. The algorithm terminates when either C or R becomes empty.

The while-loop of MRMC is executed at most N times, assuming $N > K$. SRMC is called $N \cdot K$ times during the first iteration and at most N times in each of the other $N - 1$ iterations. Thus the time complexity of MRMC is $T(MRMC) = O(N(N-1) + NK) \cdot O(N)T_B(N) = O(N^2(N+K))T_B(N)$.

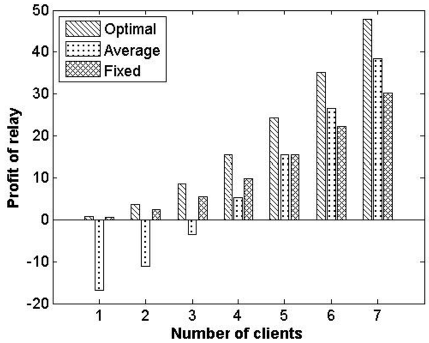
VI. Evaluations

In this section we evaluate our pricing framework by simulation study. We first consider the case of a single relay to evaluate the relay profit under the optimal client cutoff bandwidth allocation. Then we evaluate the performance of our MRMC algorithm for the case of multi-relay multi-client. Finally a public area WLAN with a relay-union is simulated to demonstrate the increase of the network utilization in terms of average bandwidth when multi-hop access via relays is employed. Note that we have performed extensive simulations over various parameter settings and achieved similar results. Due to space limitations, only a small part of the results are reported in this section.

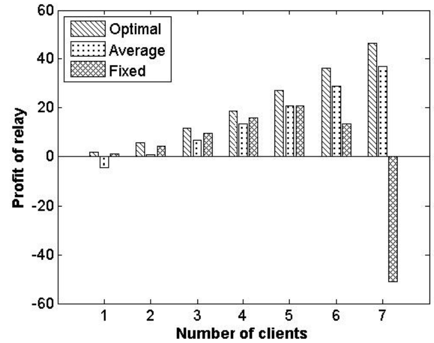
We consider the case of single-relay multi-client first. To study the impact of different cutoff bandwidth allocation strategies on the relay's profit, we vary the price/cost functions and the bandwidth demand distributions. The results for 1 to 7 clients are reported in Fig. 3, of which each data point is evaluated by only one run because the results are deterministic under the same simulation parameters. Note that no constraint on the relay bandwidth capacity and the client minimum bandwidth demand is placed in this study. The legend "Optimal" in the figures stands for the optimal cutoff bandwidth allocation obtained from Eq. (3) or Eq.(10) while "Average" and "Fixed" are listed for comparison, with "Average" meaning that 10Mbps bandwidth are equally allocated to all clients and "Fixed" meaning that the cutoff bandwidth of each client is fixed to 2Mbps.

The infinite bandwidth demand model is employed in Figs. 3(a) and 3(b). The price and cost functions are selected according to Assumptions 1 and 2. In Fig. 3(a), the price functions for the seven clients are defined by $\sqrt{B}, 3\sqrt{B}, 5\sqrt{B}, ..., 13\sqrt{B}$ while the cost function is $0.2B^2$. Fig. 3(b) employs $\log(B+1), 3\log(B+1), 5\log(B+1), ..., 13\log(B+1)$ as the price functions and $0.0004 * (2^{B+4} - 1)$ as the cost function. It is clear that our allocation scheme leads to a higher relay profit than the other two schemes. Notice that "Average" has a very bad performance when the number of clients is small because each user receives too large bandwidth. "Fixed" performs badly in Fig. 3(b) when there are 6 or 7 clients. The reason is that the exponential cost function used in 3(b) increases sharply when the relay's serving bandwidth is large, which makes the cost very high. The relay's profit may be negative for these two bad situations mentioned above, which means that the relay's cost exceeds its credit income.

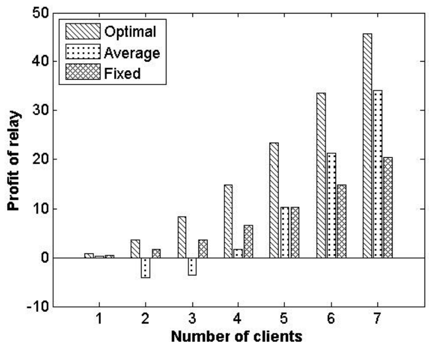
The results of the dynamic bandwidth demand model are shown in Figs. 3(c) and 3(d). The price functions are the same as in Figs. 3(a) and 3(b), and the cost functions are $0.4B^2$ in Fig. 3(c) and $0.25B^2$ in Fig. 3(d). The bandwidth demand of a client follows a uniform distribution in $[0, 5]$ Mbps in Fig. 3(c),



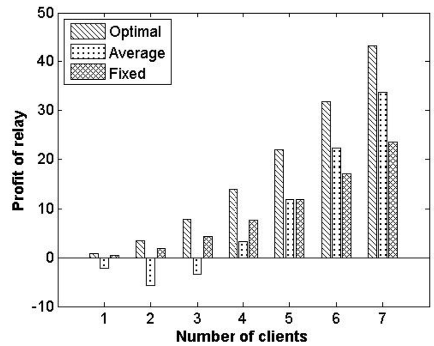
(a) Infinite model with polynomial price/cost function



(b) Infinite model with log/exp price/cost function



(c) Dynamic model with uniform distribution



(d) Dynamic model with normal distribution

Fig. 3. Relay's profit when serving multiple clients

while it follows a normal distribution with $\sigma = 2$ and $\mu = 4$ in Fig. 3(d). It can be observed that our scheme still achieves the maximum profit, while "Average" performs badly with a small number of clients and "Fixed" performs badly with a large number of clients.

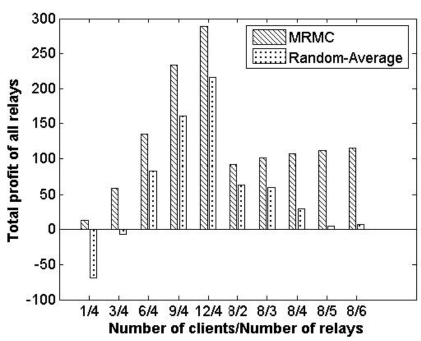


Fig. 4. Performance of MRMC

Fig. 4 depicts the simulation results of MRMC in comparison with a "Random-Average" scheme, in which clients are randomly associated with relays and relays allocate its bandwidth capacity to all clients equally. The infinite bandwidth demand model is used under the constraints on the relay bandwidth capacity and the client minimum bandwidth demand. The price functions of the clients are defined as $\sqrt{B}, 3\sqrt{B}, 5\sqrt{B}, ..., 23\sqrt{B}$ while the cost functions of the relays are $0.1B^2, 0.15B^2, 0.2B^2, ..., 0.35B^2$. The bandwidth capacities of the relays are random values on [6,14] Mbps, while the minimum bandwidth demands of the clients are random values on [0.5,2.5] Mbps. Fig. 4 reports the total profit of all relays under a variate number of relays and clients. The result of MRMC is an average over 50 runs while that of "Random-Average" is over 1000 runs to reduce randomness. It is obvious that MRMC possesses an apparent advantage over "Random-Average". The latter has very poor performance when the number of clients is small or the number of relays is large. When there are only 2 or 3 relays, the performance of these two schemes are close to each other. This is because the effect of MRMC on optimizing the relay-client association is not significant when the number of relays is small.

Finally, a small public area WLAN is simulated in which 30 users who come to the area everyday form a relay-union. On each day, these users are randomly divided into 4-8 groups around different APs. Users of each group are then randomly divided into relays and clients. The proportion of relays varies from 10% to 60% in the simulation. The relays can directly obtain 7Mbps bandwidth from their APs for free. The clients cannot directly access any AP but can purchase forwarding services from the relays. An operation of 30 days is simulated to reduce randomness. The MRMC algorithm is used for each group to compute its association and bandwidth allocation. The settings of the price functions, the cost functions, the bandwidth capacities, and the minimum bandwidth demands are the same as those for Fig. 4. We also consider the user's financial situation. Each user would like to get online for 2 hours every day. But if the user is not a relay, it has to terminate its transmissions earlier if its credits are used up. We calculate the average access bandwidths of all users over the 30-day period and report the results in Fig. 5. Here the x-axis stands for the proportion of the relays. The situation "without r-u" is considered for comparison purpose, which corresponds to the case when there is no relay-union in the network so that users who do not have direct AP accesses are not able to get online. It can be observed that the average bandwidth is increased by 40%-90% with our relay-union mechanism, which demonstrates that the users have more chances to get online by multi-hop access and the network utilization is improved under the relay-union mechanism. When the proportion of relays is low, the average bandwidth increase under the relay-union mechanism is more obvious. Thus multi-hop access is more suitable for the public area WLANs with fewer APs but a large number of users that are far from any AP.

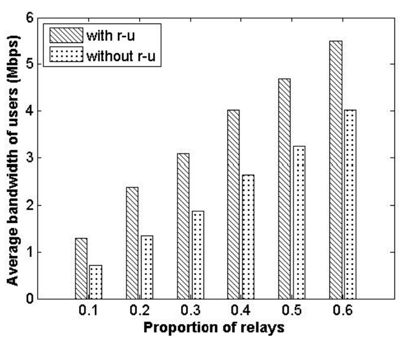


Fig. 5. Simulation results on a relay-union.

VII. CONCLUSION

This paper proposes a pricing framework for the relay-union mechanism with credit-exchange in public area WLANs. By investigating the relationship among bandwidth, charge, and profit, we present a revenue model and formulate the maximization of relay's profit as an optimal cutoff bandwidth allocation problem. We derive the solution to the problem under two different bandwidth demand models. We then extend the problem by taking the relay's capacity limitation and the client's minimum bandwidth demand into account. Finally we design the SRMC algorithm to evaluate a feasible bandwidth allocation for the single-relay multi-client scenario. We also present the MRMC algorithm to compute a relay-client association and bandwidth allocation under the multi-relay multi-client case. In the end, we report our simulation results, which demonstrates the increase of the average relay profit and the per-user bandwidth of our pricing framework.

In our future work, we propose to validate that Eq. (12) is an NP-hard problem. We will also extend our model of multi-hop access under the multi-relay multi-client scenario to a more practical one by taking AP's signal strength, load balancing, and effective multi-hop routing into consideration.

ACKNOWLEDGMENT

This work is supported by NSF of China (60911130511, 60873252), 973 Program of China (2009CB320503, 2011CB302800), and the US NSF grant CNS-0831852.

REFERENCES

[1] M. Felegyhazi, J. Hubaux, and L. Buttyan, "Nash equilibria of packet forwarding strategies in wireless ad hoc networks," *IEEE Transactions on Mobile Computing*, vol. 5, no. 5, p. 463, 2006.

[2] L. Anderegg and S. Eidenbenz, "Ad hoc-VCG: a truthful and cost-efficient routing protocol for mobile ad hoc networks with selfish agents," in *Proceedings of the 9th annual international conference on Mobile computing and networking*. ACM, 2003, pp. 245–259.

[3] L. Blazevic, L. Buttyan, S. Capkun, S. Giordano, J. Hubaux, and J. Le Boudec, "Self organization in mobile ad hoc networks: the approach of Terminodes," *IEEE Communications Magazine*, vol. 39, no. 6, pp. 166–174, 2001.

[4] L. Buttyán and J. Hubaux, "Enforcing service availability in mobile ad-hoc WANs," in *Proceedings of the 1st ACM international symposium on Mobile ad hoc networking & computing*. IEEE Press, 2000, p. 96.

[5] P. Michiardi and R. Molva, "Core: a collaborative reputation mechanism to enforce node cooperation in mobile ad hoc networks," in *Advanced communications and multimedia security: IFIP TC6/TC11 Sixth Joint Working Conference on Communications and Multimedia Security, September 26-27, 2002, Portorož, Slovenia*. Kluwer Academic Pub, 2002, p. 107.

[6] S. Buchegger and J. Le Boudec, "Performance analysis of the CONFIDANT protocol," in *Proceedings of the 3rd ACM international symposium on Mobile ad hoc networking & computing*. ACM, 2002, pp. 226–236.

[7] M. Falkner, M. Devetsikiotis, and I. Lambadaris, "An overview of pricing concepts for broadband IP networks," *IEEE Communications Surveys*, vol. 3, no. 2, 2000.

[8] D. Niyato and E. Hossain, "Competitive Pricing in Heterogeneous Wireless Access Networks: Issues and Approaches," *IEEE Network*, vol. 22, no. 6, pp. 4–11, 2008.

[9] J. Hou, J. Yang, and S. Papavassiliou, "Integration of pricing with call admission control to meet QoS requirements in cellular networks," *IEEE Transactions on Parallel and Distributed Systems*, pp. 898–910, 2002.

[10] P. Hande, M. Chiang, R. Calderbank, and J. Zhang, "Pricing under Constraints in Access Networks: Revenue Maximization and Congestion Management," in *Proc. IEEE INFOCOM*, 2010.

[11] P. Hande, M. Chiang, R. Calderbank, and S. Rangan, "Network pricing and rate allocation with content provider participation," in *Proc. IEEE INFOCOM*, 2009.

[12] L. Badia, S. Merlin, A. Zanella, and M. Zorzi, "Pricing VoWLAN services through a micro-economic framework," *IEEE Wireless Communications*, vol. 13, no. 1, pp. 6–13, 2006.

[13] Q. Wang and T. Brown, "Public safety and commercial spectrum sharing via network pricing and admission control," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 3, pp. 622–632, 2007.

[14] J. Park and M. van der Schaar, "Pricing and Incentives in Peer-to-Peer Networks," in *Proc. IEEE INFOCOM*, 2010.

[15] R. Lam, D. Chiu, and J. Lui, "On the access pricing and network scaling issues of wireless mesh networks," *IEEE Transactions on Computers*, vol. 56, no. 11, pp. 1456–1469, 2007.

[16] T. Moscibroda and S. Schmid, "On Mechanism Design Without Payments for Throughput Maximization," in *Proc. IEEE INFOCOM*, 2009.

[17] Y. Xing, R. Chandramouli, and C. Cordeiro, "Price dynamics in competitive agile spectrum access markets," *IEEE Journal on selected areas in communications*, vol. 25, no. 3, pp. 613–621, 2007.

[18] C. Saraydar, N. Mandayam, and D. Goodman, "Pricing and power control in a multicell wireless data network," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 10, pp. 1883–1892, 2001.

[19] J. Sun, E. Modiano, and L. Zheng, "Wireless channel allocation using an auction algorithm," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 5, p. 1085, 2006.

[20] J. Quirk, *Intermediate microeconomics*. Science Research Associates, 1987.